Effectful Realizability: Frameworks and Applications

Scientific Abstract

Realizability theory plays a central role in constructive foundations, both as a model-theoretic tool and as a powerful framework for analyzing and verifying the computational behavior of programs. At its core, realizability concretizes the principle of constructivity by interpreting formulas as specifications for computational entities within a programming language. However, traditional realizability models, often based on Partial Combinatory Algebras (PCAs), only support non-termination as their single computational effect, limiting their expressiveness in modeling real-world computational behaviors which typically involve computational effects such as non-deterministic computation, state manipulation, exceptions, continuations etc.

The overall aim of the proposed study is to introduce a foundational extension to realizability theory through the integration of effectful computation into realizability, and investigate the implications of this integration on the resulting effectful theories. While there is significant research into modeling and reasoning about effectful computation, we aim to *internalize* and utilize effectful computation within the system itself, by enabling realizers—the computational entities that validate formulas—to embody these effects. This means that our framework will support the internalization and uniform treatment of computational effects directly within realizability models, which is crucial for the development of robust and expressive computational theories and for reasoning about them collectively.

The project develops effectful realizability from two complementary perspectives: algebraic and syntactic. Algebraically, we will develop a novel generalization of PCAs that incorporates monadic structures to encapsulate diverse computational effects in a rigorous way. Syntactically, we will develop a higher-order program logic that enables reasoning about effectful realizers and supports a syntactic translation from Higher-Order Logic to effectful computational models, facilitating the systematic extraction of effectful realizers from proofs. Third, we will apply these frameworks to foundational principles in constructive mathematics, such as Choice and Continuity principles, analyzing how computational effects may effect the validity of these principles and offer novel computational interpretations.

The proposed research will push forward the state-of-the-art of realizability theory, enriching both verification systems and constructive foundations. As realizability serves as the basis for many constructive systems used for mechanical verification, supporting general effects would enable the internal development of mechanically verified software utilizing parallelism, concurrency, probability, state, exceptions, and other constructs essential to modern to large-scale efficient software systems. From a foundational point-of-view, effectful realizability will broaden many constructive foundations which, in turn, can be exploited to provide novel computational meaning to foundational principles.